An Original Approach to the Design of Bandpass Cavity Filters with Multiple Couplings

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Abstract— **A novel approach to the design of general cavity filters with each cavity coupled to an arbitrary number of other cavities is presented. This approach is based on a suitable characterization of the filter structure which does not require to model separately the cavities (resonators) and the coupling elements. Suitably defined equivalent admittances are associated with each cavity allowing to design the filter structure once the parameters of a suitable low-pass prototype are given; an efficient procedure for the synthesis of such a prototype with equiripple passband response is also presented which allows to arbitrarily prescribe transmission zeros placed in the complex plane (even asymmetrically). The described design approach is particularly convenient when the filter structure does not allow a simple modeling of the resonators and coupling elements separately. This is the case of slot-coupled cavity filters and of filter structures based on arrays of coupled transmission lines. It is also shown that the simplified design approach often adopted in the past, where only two coupled cavities at a time are considered, can produce large errors even in the case of filters with all attenuation poles at infinity (i.e., two couplings per cavity).**

Index Terms— **Bandpass filters, cavity resonator filters, microwave filters.**

I. Introduction

The design of direct coupled bandpass cavity microwave filters is a matter widely investigated in the literature. The basic approach proposed in the pioneering works of [1]–[3] (mainly in the area of waveguide filters) is still adopted for the practical realization of several filter structures. Originally most of the works discussed the design of filters with all attenuation poles at infinity, but in the following years several authors have investigated possible design approaches for microwave filters also with transmission zeros at finite frequencies. In particular, [4]–[6] proposed the use of multiple-coupled cavity configuration for realizing such zeros; these works, however, are mainly concerned with the network synthesis problem (i.e., the evaluation of the coupling coefficients between the cavities in order to obtain the desired transmission characteristic). More recently, other works have investigated the synthesis of general low-pass prototypes with transmission zeros suitable for direct-coupled cavity design [7]–[9].

It is important to observe that once the coupling coefficients required between the cavities have been evaluated with one of the above methods the design of a filter following the original Cohn's approach (with or without transmission zeros) requires a separate modeling of both of the cavities (through equivalent resonators) and the coupling structures (generally by means of impedance or admittance inverters); moreover, it is necessary that the equivalent parameters of the coupling structures and resonators are independent of each others. There are, however, some cases (depending on the specific filter structure considered) in which either the separate modeling of the coupling structures and the resonators cannot be performed with accuracy or there is a strong interaction between these elements (the first situation happens, for instance, with closed metal cavities coupled by means of slots in the common walls; the second one is typical of coupled-line filters). In the case of slot-coupled cavities it is usual to design the filter according to the following simplified approach [4], [10], [11]:

1) coupling coefficients evaluated with one of the aforesaid methods;
2) cavities dimensioned with all the couplings removed;
3) geometrical dimensions of the coupling structures determined by considering a single coupling at a time (i.e., two cavities coupled through the coupling structure considered, discarding all the couplings with the other cavities).

It can then be observed that although this approach allows the use of highly accurate field-based numerical methods (i.e., mode matching, finite elements, integral equations, etc.) for evaluating the geometrical dimensions of the coupling structures there is an intrinsic inaccuracy due to discarding for each cavity pair all of the couplings but one. This inaccuracy concerns the dimensioning of both the coupling elements and the cavities; however, the small errors in the tuning frequency of the cavities can be easily compensated during the filter alignment, while those on the coupling structures require long and expensive cut-and-try experimental adjustments for the proper filter operation.

In this paper, a general approach for designing multiple-coupled cavity filters is presented, which overcomes the aforesaid drawbacks; this approach, applicable to a large class of specific filter structures, is based on the introduction of suitable equivalent cavities which include the effect of the coupling elements. The unknown geometrical dimensions of the coupling structures for a given set of the coupling coefficients values are evaluated by solving a nonlinear system with an efficient numerical method. A novel method is also presented for evaluating the coupling coefficients when transmission zeros are required (either purely imaginary, complex, or even frequency asymmetric). This method is based on the synthesis of a general low-pass prototype with an equiripple passband response.

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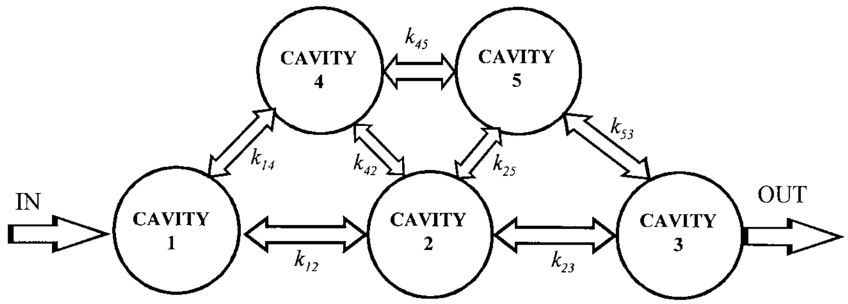

Fig. 1. General configuration of multicoupled cavity filters. Each cavity can have an arbitrary number of couplings with other cavities (represented in the figure by the coupling coefficients k_{mn}).

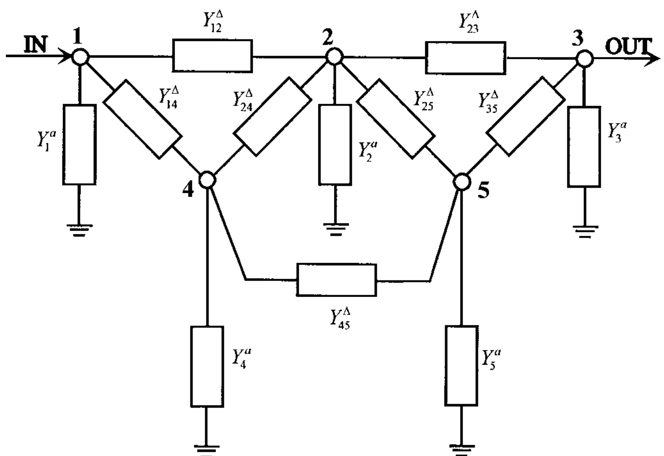

Fig. 2. Equivalent circuit of the general filter configuration in Fig. 1. Nodes 1, 2, 3, $\cdots$ represent suitable reference sections of the cavities.

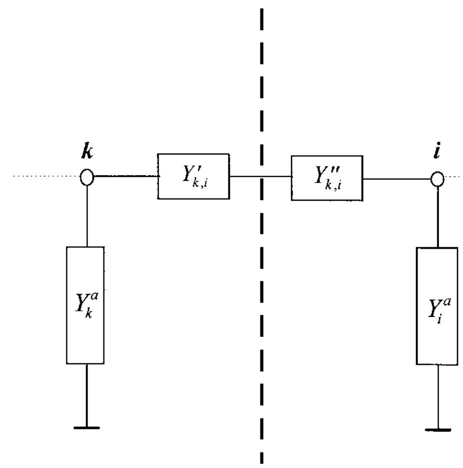

Fig. 3. Splitting of $Y_{k,i}^{\Delta}$ into two series-connected admittances $Y'_{k,i}$ and $Y''_{k,i}$; insertion at the junction node of a short circuit (electric wall) or an open circuit (magnetic wall).

To give evidence of the effects arising from discarding multiple couplings during the design, the numerical simulations on some test filters (designed with both the simplified approach and the procedure proposed here) have been performed and the results obtained are discussed in the paper.

II. Fundamentals of the Design

A. Modeling

The general filter structure in Fig. 1 can be represented through the $N \times N$ admittance matrix $\boldsymbol{Y_t}$ defined at the reference ports of the N resonators. A possible circuit model of the structure is given in Fig. 2, where the admittances Y_i^a and $Y_{i,j}^{\Delta}$ are dependent on the elements of $\boldsymbol{Y_t}$ through the following relationships:

$$Y(i,j) = -Y_{i,j}^{\Delta} \tag{1a}$$

$$Y(i,i) = Y_i^a + \sum_{k \neq i} Y_{k,i}^{\Delta} = \overline{Y}_i^a + jB_{T,i} + \sum_{k \neq i} Y_{k,i}^{\Delta}. \tag{1b}$$

In (1b) the admittance Y_i^a has been split into two parts in order to introduce $jB_{T,i}$ which represents the tuning susceptances required for the practical alignment of the filter (they are generally realized by means of metallic screws); the summations in (1b) are performed over all the coupling admittances $Y_{k,i}^{\Delta}$ connected at node i. Note that $Y(i,i)$ represents the equivalent resonator modeling the ith resonant cavity.

Let consider the kth cavity with all the couplings to the neighboring cavities: each coupling admittance $Y_{k,i}^{\Delta}$ can be arbitrarily split into two series connected admittances $Y'_{k,i}$ and $Y''_{k,i}$ (Fig. 3). Introducing open circuits or short circuits at the junction points between these admittances, the following equivalent admittances can be defined:

$$\begin{aligned} Y_{k(S,i)}^t &= \overline{Y}_k^a + Y'_{k,i} + jB_{T,k} = \overline{Y}_{k(S,i)}^t + jB_{T,k} \\ Y_{k(O)}^t &= \overline{Y}_k^a + jB_{T,k} = \overline{Y}_{k(O)}^t + jB_{T,k} \end{aligned} \tag{2a}$$

$$\begin{aligned} Y_{i(S,k)}^t &= \overline{Y}_i^a + Y''_{k,i} + jB_{T,i} = \overline{Y}_{i(S,k)}^t + jB_{T,i} \\ Y_{i(O)}^t &= \overline{Y}_i^a + jB_{T,i} = \overline{Y}_{i(O)}^t + jB_{T,i}. \end{aligned} \tag{2b}$$

From a physical point of view $Y_{k,(O)}^t$ represents the equivalent admittance of the kth cavity with magnetic walls inserted inside the coupling structures at the sections corresponding to the junction points of $Y'_{k,i}$ and $Y''_{k,i}$. $Y_{k,(S,i)}^t$ is the equivalent admittance of the kth cavity with an electric wall inserted inside the coupling structure between cavities k and i, and magnetic walls inserted inside all the other coupling structures.

It can be observed that independently of the specific localization of the sections where magnetic or electric walls are inserted, the coupling admittances $Y_{k,i}^{\Delta}$ can be expressed as a function of $Y_{k,(O)}^t$ and $Y_{k,(S,i)}^t$ through the following relationship:

$$Y^{\Delta}_{k,i} = \left\{ \frac{1}{Y^t_{k(S,i)} - Y^t_{k(O)}} + \frac{1}{Y^t_{i(S,k)} - Y^t_{i(O)}} \right\}^{-1} = \left\{ \frac{1}{\overline{Y}^t_{k(S,i)} - \overline{Y}^t_{k(O)}} + \frac{1}{\overline{Y}^t_{i(S,k)} - \overline{Y}^t_{i(O)}} \right\}^{-1}. \quad (3)$$

Equation (3) has been obtained by deriving $Y'_{k,i}$ and $Y''_{k,i}$ from (2a) and (2b) and taking into account that $Y^{\Delta}_{k,i}$ is the series of $Y'_{k,i}$ and $Y''_{k,i}$; note also that $Y^{\Delta}_{k,i}$ is independent on the tuning susceptances $B_{T,k}$ and $B_{T,i}$.

Using (2) and (3), all the elements of the overall admittance matrix $\boldsymbol{Y_t}$ can be computed as a function of the equivalent admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$, which, in turn, can be derived through the analysis of each physical cavity suitably terminated (by means of electric and/or magnetic walls introduced inside the coupling elements). This allows to take into account in the computation of the parameters of the equivalent resonators necessary for the filter design the effect of all the couplings outgoing from each physical cavity. In this way, the overall accuracy of the design can be very high depending only upon the degree of accuracy of the evaluation of the admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$. Note that very often the practical dimensioning of the coupling structures in coupled-cavity filters is performed by discarding for each pair of coupled resonators all couplings but one [4]. This approach makes the design of the filter relatively simple but it may require relevant experimental adjustments, especially in case of moderate bandwidth with transmission zeros close to the passband.

It can be observed that although $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$ depend on the actual localization of the electric and magnetic walls introduced inside the coupling susceptances, $Y^{\Delta}_{k,i}$ is independent on this choice. The sections where it is convenient, from a computational point of view, to place the electric and/or magnetic walls (for the numerical evaluation of $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$) are dependent on the specific filter structures; for example, in the case of cavity resonators coupled through thin diaphragms or slots, the right placement of the electric or magnetic walls is just in the sections of the coupling elements; in the case of comb filters, the electric or magnetic walls can be conveniently located midway between the coupled line resonators.

B. Development of a General Design Procedure

It is known that the coupling between resonators k and i in the direct-coupled cavities' filters can be expressed by means of the coefficient $k_{k,i}$ defined as [12]

$$k_{k,i} = - \left. \frac{\left| Y^{\Delta}_{k,i} \right|}{\sqrt{B_{eq,k} \cdot B_{eq,i}}} \right|_{\omega=\omega_0} \quad (4)$$

where $\omega_0 = 2\pi f_0$ is the midband angular frequency of the filter and $B_{eq,k}$ is the susceptance slope parameter of the kth cavity; in absence of losses one gets

$$B_{eq,k} = \frac{1}{2}\omega_{0,k} \left. \frac{\partial |Y(k,k)|}{\partial \omega} \right|_{\omega=\omega_{0,k}} \quad (5)$$

where $\omega_{0,k}$ is the resonance angular frequency of the kth cavity.

Equation (4) can then be evaluated as a function of the equivalent admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$ by using (2), (3), and (5). Once the numerical values of the coupling coefficients $k_{i,j}$ have been obtained (for instance, by means of the normalized low-pass prototype presented in Appendix A) the system of nonlinear equations given by (6), at the bottom of the page, must be solved in order that the actual filter structure approximates the electrical response of the de-normalized prototype.

In (6) the summations are performed over all resonators k coupled to i (j). In addition, the resonance of each cavity must also be imposed from (7a), taking into account (2) the tuning susceptances $B_{T,i}$ can be directly derived ((7a) and (7b) shown at the bottom of the next page). Equations (6) and (7) generally depend, through the equivalent admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$, on all the geometrical and electrical parameters which define the specific filter structure considered; however, a number of these parameters are, in general, preliminarily assigned according to some suitable criteria (minimization of the losses, reduction of the overall volume, optimization of the out-of-band behavior, and so on); also, the geometrical dimensions of the cavities may be imposed *a priori* (then determining the values of the susceptances $\overline{Y}^a_i$). The design procedure presented here assumes, as design unknown, only the geometrical parameters which determine the couplings between the cavities; moreover, the number of these parameters is assumed to be equal to the overall number of couplings m_{tot}.

The design can then be performed by numerically solving the system (6) of m_{tot} nonlinear equations, as a function of the geometrical coupling parameters of the specific filter structure; during the solution of the system, given a set of unknowns values, the resonance condition (7b) determines the values of the tuning susceptances $B_{T,i}$ [required for evaluating the left-hand side (LHS) of (6)].

$$k_{i,j} + \left. \frac{2\left\{ \frac{1}{Y^t_{i(S,j)} - Y^t_{i(O)}} + \frac{1}{Y^t_{j(S,i)} - Y^t_{j(O)}} \right\}^{-1}}{\omega_0 \sqrt{\frac{\partial}{\partial\omega}\left\{ Y^t_{i(O)} + \sum_k \left\{ \frac{1}{Y^t_{i(S,k)} - Y^t_{i(O)}} + \frac{1}{Y^t_{k(S,i)} - Y^t_{k(O)}} \right\}^{-1} \right\} \cdot \frac{\partial}{\partial\omega}\left\{ Y^t_{j(O)} + \sum_k \left\{ \frac{1}{Y^t_{j(S,k)} - Y^t_{j(O)}} + \frac{1}{Y^t_{k(S,j)} - Y^t_{j(O)}} \right\}^{-1} \right\}}} \right|_{\omega=\omega_0} = 0 \quad (6)$$

The numerical solution of the nonlinear system is based on the minimization of a cost function (nonlinear least squares optimization). A very efficient algorithm is represented by the Gauss–Newton Method which has been employed for designing the test filters considered in the following section. In spite of the strong nonlinearity of the system to be solved, acceptable solutions have always been found for all the designs performed.

C. Expression of the Coupling Coefficients as a Function of the Resonance Frequencies of the Equivalent Cavities

It is known that the coupling coefficients between two coupled resonators can be given as the function of two resonance frequencies related to suitably defined equivalent resonators [11]. It has been found also that in the general case considered in this paper (array of resonators with multiple couplings) the coupling coefficients can be expressed as the function of the resonant frequencies of the equivalent admittances $Y^t_{m,(O)}$ and $Y^t_{m,(S,n)}$ through the following relationship:

$$k_{m,n} = \frac{\frac{1}{f^2_{m,(S,n)}+f^2_{m,(O)}} + \frac{1}{f^2_{n,(S,m)}+f^2_{n,(O)}}}{\frac{1}{f^2_{m,(S,n)}-f^2_{m,(O)}} + \frac{1}{f^2_{n,(S,m)}-f^2_{n,(O)}}} \tag{8}$$

where $f_{m,(S,n)}$ and $f_{m,(O)}$ are the resonant frequencies of $Y^t_{m,(S,n)}$ and $Y^t_{m,(O)}$, respectively. Note that (8) has not been analytically derived and so it cannot be stated that it exactly holds; however, the accuracy has been verified by applying (8) for computing the coupling coefficients in several test filters (designed with the method presented here) and comparing these values with those originally obtained from the synthesis of the low-pass prototype.

III. Verification of the Design Procedure

The aim of this section is to show the improvement in the design accuracy obtainable through the novel approach with respect to the evaluation of each coupling element one at a time (discarding all the others). Some test filters have been designed following both approaches and the simulated responses obtained in the two cases have been compared in order to emphasize the effects produced by discarding the presence of multiple couplings in the design.

It is well worth observing that the overall accuracy in the design of a generic filter structure (including those considered here) depends both on the aspects concerning the multiple couplings (to which this paper is devoted) and also on the accuracy of the modeling of the resonators; this means that the overall design accuracy is affected, in the novel approach, by the numerical evaluation of the equivalent admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$.

However, this aspect of accuracy (i.e., the cavity modeling) is common to whatever approach is followed in the design of coupled-cavities' filters. The most accurate methods are based on the numerical evaluation of the electromagnetic field inside the cavities (mode-matching method, integral equation technique, spectral domain approach, and so on); these methods, however, are generally not suitable for filter design procedures (as that presented here) because of the very long computer time they typically require. For design purposes, a less accurate but more viable choice consists in the representation of the cavities by means of simple equivalent circuits suitably defined. In the case of overall unsatisfactory accuracy, optimization procedures may be employed after the design using a more sophisticated model for the cavities' characterization (note that the direct optimization without a preliminary design gives generally unsatisfactory results, especially for narrow-band filters with transmission zeros close to the passband).

In the following (since the focus of this paper is not on the accuracy of the cavity modeling), a simple equivalent circuit has been adopted for computing the admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$; however, it is worth noting again that the improvement of accuracy in the test filters design determined by taking into account the multiple couplings in each cavity (which is the matter investigated here) can be demonstrated as well.

A. Test Filter Structure

The filter structure chosen for testing the novel design approach is constituted by the generalized slot-coupled combline filter [13]; this structure is of particular interest in mobile communications applications, allowing the construction of high-selectivity, phase-equalized filters.

The structure under consideration is shown in Fig. 4. The basic resonator is constituted by a circular metallic rod of diameter d and length BL placed inside a rectangular cavity having the cross-section dimensions a and b, respectively. The rod is short-circuited at one end and resonates with a capacitive susceptance (generally realized through a metallic screw) at the other end. The cavities are displaced in the structure in two parallel rows (folded configuration) in order to allow couplings between nonadjacent resonators. The couplings are realized by removing part of the walls which separate adjacent cavities (each resonator can have at most three couplings in this configuration). The input and output ports are of a coaxial type and are coupled to the rod in the first and last cavities by means of taps at a suitable distance from the short circuit (the design of the taps is not considered in this work).

$$Y(i,i)|_{\omega=\omega_{0,i}} = \left\{ Y^t_{i(O)} + \sum_k \left\{ \frac{1}{Y^t_{i(S,k)} - Y^t_{i(O)}} + \frac{1}{Y^t_{k(S,i)} - Y^t_{k(O)}} \right\}^{-1} \right\}_{\omega=\omega_{0,i}} = 0 \tag{7a}$$

$$jB_{T,i}(\omega_{0,i}) = -\left\{ \overline{Y}^t_{i(O)} + \sum_k \left\{ \frac{1}{\overline{Y}^t_{i(S,k)} - \overline{Y}^t_{i(O)}} + \frac{1}{\overline{Y}^t_{k(S,i)} - \overline{Y}^t_{k(O)}} \right\}^{-1} \right\}_{\omega=\omega_{0,i}} \tag{7b}$$

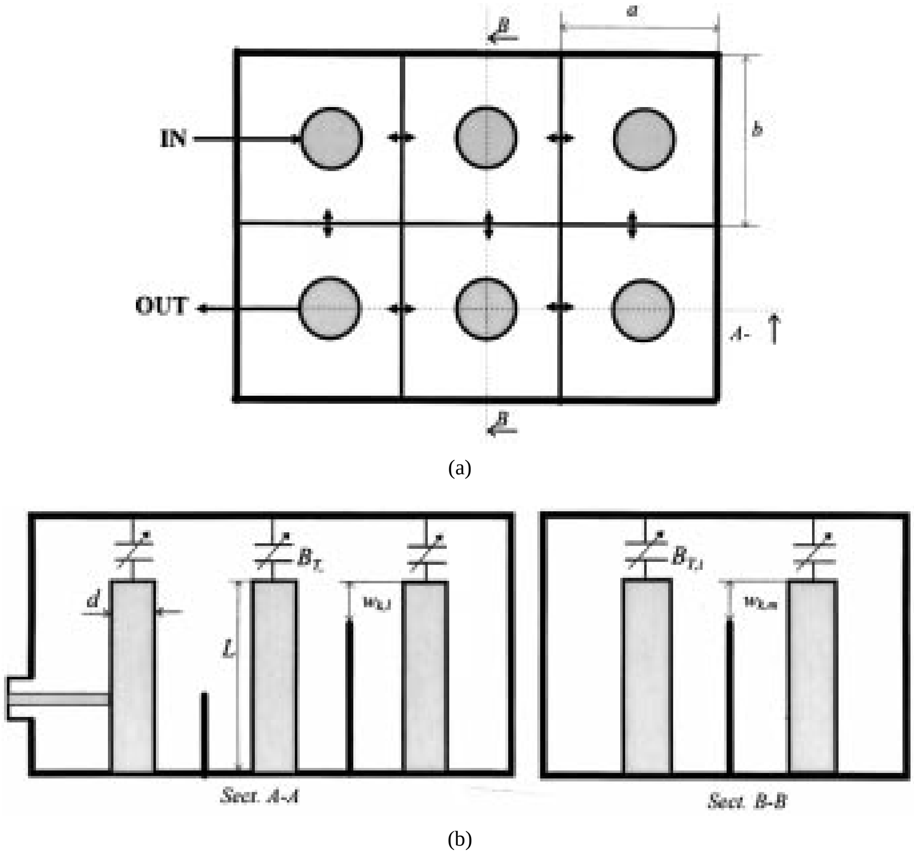

Fig. 4. Schematization of the generalized slot coupled combline filters. (a) Top view. (b) Side views. The arrows in the top view show all the possible couplings for the considered structure.

The design unknown parameters are represented here by the length w_k of the slots (the walls are removed starting from the open side of the rods); all the other geometrical parameters must be preliminary assigned according to suitable criteria. For instance, when negative couplings are required, the length L must be larger than 45° (the usual value adopted in a comb filter). A typical choice in this case is between 60°–80°.

The cross-section dimensions a and b must be chosen sufficiently small to allow the realization of the largest coupling coefficients (positive and negative) obtained from the electrical synthesis. Another aspect to be taken into account in the choice of the above parameters is the unloaded Q factor of the cavities (which determine the overall losses of the filter).

Note, however, that the aspects concerning the choices for the optimum design of generalized comb filters are not faced in this paper (the aforementioned aim of which is the investigation of the effects produced by multiple couplings in the cavity filter's design).

In accordance with this aim the admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$ related to the filter structure described above, are modeled by means of a simple equivalent circuit, which is described in detail in Appendix B. This model is adopted for both the design and analysis of the filter; in particular, the frequency response is obtained through the following steps:

1) computation of the overall admittance matrix $\boldsymbol{Y_t}$ using (1)–(3) (Section IIA), taking into account the frequency dependence of all parameters;
2) computation of input–output impedance matrix $\boldsymbol{Z}$ of the filter (order 2) by inverting the matrix $\boldsymbol{Y_t}$ (order N) and extracting the elements $(1,1)$, $(1,N)$, $(N,1)$, (N,N), where N is the number of filter cavities;
3) computation of the filter response (attenuation) vs frequency from matrix $\boldsymbol{Z}$.

During the analysis, the coupling of the external loads with the first and last resonators is described by means of ideal transformers.

B. Results from the Test Filter

Electrical requirements typical of radio mobile applications are assumed for the test filter. It is constituted by six cavities and it shows a passband equiripple response, with a pair of imaginary transmission zeros imposed for improving selectivity and a pair of complex zeros imposed for equalizing the passband group delay response. Table I shows the values assigned to the filter design parameters together with the transmission zeros' locations obtained from a numerical search procedure. The synthesis of the normalized low-pass prototype (following the procedure in Appendix A) gives the coupling coefficients, also specified in Table I; note that being the transmission zeros symmetrically located around the passband, the prototype does not contain the couplings between resonators k and $N-k$ (the generalized comb structure considered here is then suitable for the filter realization).

TABLE I
ELECTRICAL SPECIFICATION AND COUPLING COEFFICIENTS OF THE TEST FILTER

Electrical Parameter	*Value*
Midband Frequency (MHz)	917.5
Bandwidth (MHz)	3.4
Return Loss in band (dB)	20.8
Number of resonators	6
Attenuation poles (MHz)	j914, j921.01
Equalization zeros (MHz)	±1.252 + j 917.5
Coupling Coefficient k_{12}	+0.00321
Coupling Coefficient k_{23}	+0.00227
Coupling Coefficient k_{34}	+0.00175
Coupling Coefficient k_{45}	+0.00227
Coupling Coefficient k_{56}	+0.00321
Coupling Coefficient k_{16}	-0.00016
Coupling Coefficient k_{25}	+0.00060

TABLE II
PRELIMINARY ASSIGNED GEOMETRICAL PARAMETERS.

Geometrical Parameter	*Value (mm)*
Cavity width b	50
Cavity height a	60
Rods diameter d	30
Rods length L	50

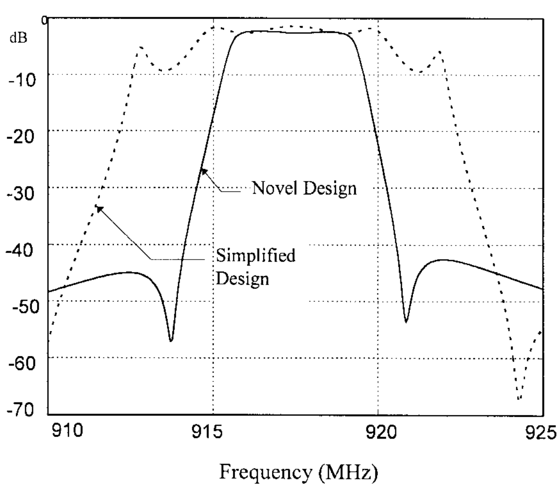

Fig. 5. Comparison between the frequency responses of the test filter (with the transmission zeros) designed according to the new procedure (continuous line) and the simplified old approach (dashed line).

In Table II the preliminary geometrical dimensions assigned to the resonators are reported; note that the electrical resonator's length is large enough (55°) to allow the realization of the negative coupling k_{16}.

The evaluation of the coupling slots' length w_k has been performed by following both the design approaches considered here (i.e., the novel one proposed in this paper and the simplified one recalled in the Introduction). The values of w_k obtained in the two cases are shown in Table III; it can be seen that the differences are relatively large. The inaccuracy of the simplified design approach is evident by examining the frequency response (attenuation) of the two designed filters reported in Fig. 5; note that the tuning susceptances in the simplified design have been computed on the actual filter structure (i.e., taking into account all the couplings between the cavities). This means that the degradation of the filter response is only due to neglecting the multiple couplings during the design and not to mistuning effects.

TABLE III
SLOTS LENGTH COMPUTED WITH THE NOVEL PROCEDURE AND WITH THE SIMPLIFIED APPROACH, WHICH DISCARDS THE MULTIPLE COUPLINGS

Slots	*Novel Design*	*Simplified Design*
$w_{12}=w_{56}$	19.79 mm	21.27mm
$w_{23}=w_{45}$	14.14mm	19.67mm
w_{34}	12.67mm	18.72mm
w_{16}	14.50 mm	14.23mm
w_{25}	10.11 mm	16.27mm

TABLE IV
SLOTS' LENGTHS AND COUPLING COEFFICIENTS FOR THE ALL POLES TEST FILTER

Slots	$k_{i,i+1}$	*Novel Design*	*Simplified Design*
$w_{12}=w_{56}$	0.00318	19.74 mm	21.21 mm
$w_{23}=w_{45}$	0.00228	13.97 mm	19.70 mm
w_{34}	0.00218	13.50 mm	19.51 mm

Examining the results shown above, one could attribute the large differences resulting from the two design approaches to the presence of the two additional couplings required for realizing the transmission zeros. As a matter of fact, this is not completely true. In fact, even if any transmission zero is required, there are two couplings for each cavity. So, if the slot length realizing one of these couplings is computed by disregarding the presence of the other, the same inaccuracy previously found will result.

This has been verified with the above test filter by removing the couplings 1–6 and 2–5; the resulting six-poles Chebychev filter has been designed again following the two approaches investigated here and the results (the coupling slots' lengths) are reported in Table IV, together with the required coupling coefficients.

As can be seen from Table IV, the differences in the slots' lengths values obtained from the two design approaches are similar to those obtained when the transmission zeros were present. The simulated frequency response of the filter (Fig. 6) also confirms the previous considerations. Concluding this section, it is advisable to again remark that the purpose pursued here has not been to develop an accurate design procedure for slot-coupled combline filters, but to show that discarding multiple couplings during the filter design may produce relevant errors; however, the discrepancies shown here between exact and approximate design cannot be generalized to whatever filter configuration (in fact, other structures may present far less pronounced differences).

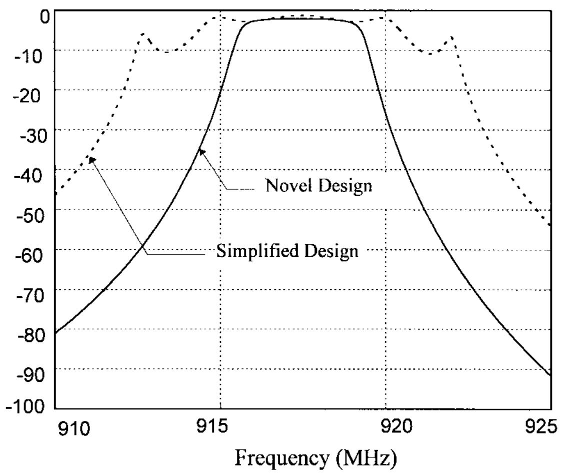

Fig. 6. Frequency behavior obtained with the two design approaches when the test filter has only two couplings per cavity (Chebychev response).

IV. Conclusion

A novel approach to the design of cavity filters with multiple couplings between the cavities has been presented. The method is based on the definition of suitable equivalent uncoupled cavities obtained by introducing electric and/or magnetic walls inside the coupling structures. The dimensioning of the coupling structures is performed through the numerical solution of a nonlinear system of equations expressing the requirements imposed by the coupling coefficients values and by the resonance condition. The values of the coupling coefficients (and of the tuning frequency of each resonator) are obtained through the synthesis of a low-pass prototype, which allows the imposition of an arbitrary frequency response even with asymmetric transmission zeros. By means of a test filter, the improvement in the design accuracy has been demonstrated (with respect to a simplified design approach which considers a single coupling at a time).

Appendix A
General Equiripple Low-Pass Prototype

The general low-pass prototype is synthesized in the form of the cross-coupled network in Fig. 7; in order to also allow an asymmetric response, frequency invariant susceptances are introduced in parallel to each normalized capacitance. It is known [14] that a similar network allows to obtain in the prototype transfer function arbitrary transmission zeros (both complex and imaginary) with an equiripple response in passband.

The transfer function is represented by parameter $S_{21}(s)$, where $s = \Theta + j\Omega$ is the normalized complex frequency. The

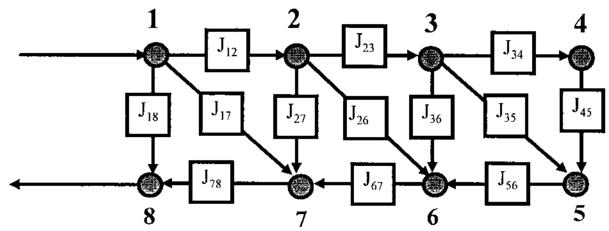

Fig. 7. General cross-coupled prototype network. Each node represents a capacitance g_i in parallel with a frequency invariant susceptance b_i; the parameters J_{ij} are the admittance inverters between nodes i and j.

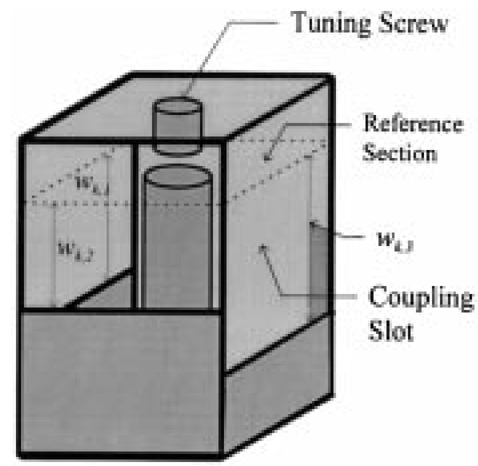

Fig. 8. Relevant geometric parameters of the generic ith cavity inside the slot-coupled combline filter. The reference section is located at the end of the round rod.

attenuation characteristic can then be expressed as

$$A(\Omega^2) = \frac{1}{|S_{21}(j\Omega)|^2} = 1 + \varepsilon^2 C_N^2(\Omega) \qquad \text{(A1)}$$

where ε depends on the required passband ripple and C_N is a generalized Chebychev function expressible as a ratio of two polynomials

$$C_N(\Omega) = \frac{N(\Omega)}{D(\Omega)} = \frac{\prod_k^{1,N}(\Omega - \Omega_{d,k})}{\prod_k^{1,Nz}(\Omega - \Omega_{z,k})} \qquad \text{(A2)}$$

where N is the order of the filter (number of poles), N_z is the number of transmission zeros, whose values are $j\Omega_{z,k}$, and $j\Omega_{d,k}$ are the reflection zeros. Note that $\Omega_{d,k}$ are real numbers because the reflection zeros must be on the imaginary axis. The transmission zeros $j\Omega_{z,k}$ are in general complex numbers which must be symmetrically prescribed with respect to the imaginary axis in order to synthesize the low-pass prototype as the cross-coupled network in Fig. 7.

$$C_N(\Omega) = \begin{cases} \cos\left[(N - N_z)\cos^{-1}(\Omega) + \sum_k^{1,Nz} \left|\Re\left\{\cos^{-1}\left(\frac{1-\Omega\cdot\Omega_{z,k}}{\Omega-\Omega_{z,k}}\right)\right\}\right|\right], & |\Omega| \le 1 \\ \cosh\left[(N - N_z)\cosh^{-1}(\Omega) + \sum_k^{1,Nz} \left|\Re\left\{\cosh^{-1}\left(\frac{1-\Omega\cdot\Omega_{z,k}}{\Omega-\Omega_{z,k}}\right)\right\}\right|\right], & |\Omega| > 1 \end{cases} \qquad \text{(A3)}$$

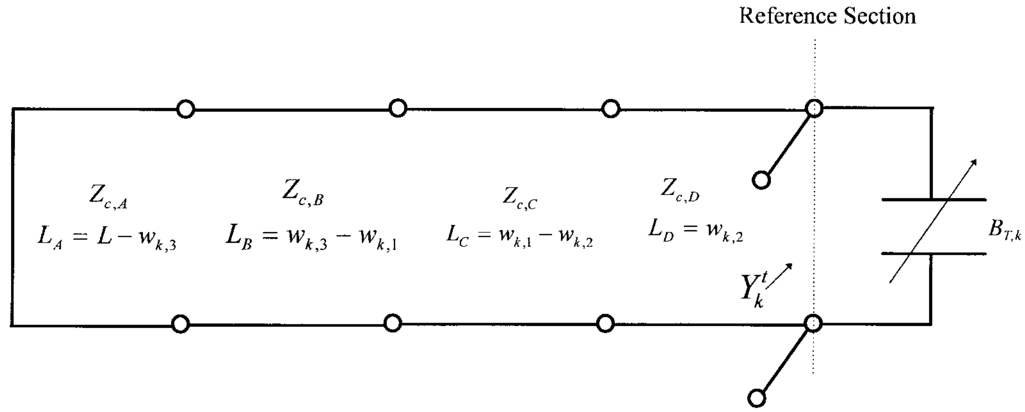

Fig. 9. Equivalent circuit of the cavity represented in Fig. 8. Each transmission line section is defined by the characteristic impedance $Z_{c,x}$ and by the length L_X. The admittance Y_k^tmay represent either $Y_{k,(0)}^t$ or $Y_{k,(S,i)}^t$ depending on the electric and/or magnetic walls located at the slots' sections.

For determining the unknown reflection zeros $j\Omega_{d,k}$ a closed form expression for $C_N(\Omega)$ must be available. It has been found that the expression proposed by [8] for imaginary transmission zeros works as well also for complex zeros if given in the following form (see (A3) at the bottom of the previous page). Once the zeros $j\Omega_{z,k}$ have been imposed, the polynomial $N(\Omega)$ at $\Omega = \Omega_i$ can be expressed as

$$N(\Omega_i) = C_N(\Omega_i) \cdot \prod_{k}^{1,Nz} (\Omega_i - \Omega_{z,k}) = 1 + \sum_{k}^{1,N} a_k \cdot \Omega_i^k. \tag{A4}$$

The coefficients a_k can be computed with a polynomial fitting procedure by selecting some values of Ω_i in the range (–1, 1) and computing the corresponding $C_N(\Omega_i)$ values with (A3). The selection of the normalized frequencies Ω_i is not critical, even if the order of the filter is high; however, using a nonuniform distribution of the frequency points (with an increasing number toward the passband edges) the computations become faster and more accurate. Once the coefficients a_k are known, the reflection zeros $j\Omega_{d,k}$ are readily determined and the poles of the transfer function can be obtained by building up the polynomials $P(s)$ and $F(s)$, which define the scattering parameters S_{21} and S_{11} in the complex plane s

$$S_{21} = \frac{P(s)}{E(s)}, \qquad S_{11} = \frac{F(s)}{E(s)}. \tag{A5}$$

Note that the roots of $P(s)$ are $j\Omega_{z,k}$, the roots of $F(s)$are $j\Omega_{d,k}$, and the poles of the transfer function are the roots of $E(s)$. These can be obtained from the polynomial $E(s) \cdot E(-s)$, defined as

$$E(s) \cdot E(-s) = \begin{cases} [P(s) + jF(s)] \cdot [P(s) - jF(s)] & (N \text{ even}) \\ [P(s) + F(s)] \cdot [P(s) - F(s)] & (N \text{ odd}) \end{cases} \tag{A6}$$

and selecting the roots of $E(s) \cdot E(-s)$ with the negative real part.

Once the transfer polynomials $P(s)$, $F(s)$, and $E(s)$ have been determined the synthesis of the prototype low-pass can be carried out using the procedure described by [14] based on the chain matrix description of the normalized network loaded with unitary resistances. At the end of the synthesis procedure the values of the prototype parameters g_k, b_k, and$J_{mn}$are then available.

The denormalized passband filter is obtained by substitution of the capacitance g_k and the susceptances b_k with equivalent parallel resonators having the following parameters:

$$f_{res,k} = f_0 \cdot \left\{ \sqrt{\left(\frac{B}{f_0} \cdot \frac{b_k}{2g_k}\right)^2 + 1} - \left(\frac{B}{f_0} \cdot \frac{b_k}{2g_k}\right) \right\}$$
$$\frac{B_{eq,k}}{Y_0} = \frac{1}{2} \frac{g_k}{B/f_0} \left(1 + \frac{f_0^2}{f_{res,k}^2}\right) \tag{A7}$$

where $f_{res,k}$ and $B_{eq,k}$ represent the resonant frequency and the susceptance slope parameter of kth equivalent resonator, respectively. f_0 is the midband filter frequency, B is the filter bandwidth, and Y_0 is an arbitrary reference admittance.

The load and generator resistances of the denormalized filter are given by

$$R_L = R_G = \frac{(f_0/B)g_1 + b_1}{B_{eq,1}}. \tag{A8}$$

Finally, the coupling coefficient k_{mn} between resonators m and n is obtained through

$$k_{mn} = \frac{J_{mn}/Y_0}{\sqrt{(B_{eq,m}/Y_0) \cdot (B_{eq,n}/Y_0)}}. \tag{A9}$$

Note that the frequency response of the denormalized passband filter around f_0 does not exactly follow that of the normalized prototype around the origin due to the presence in the prototype of the frequency invariant susceptances; however, the approximations introduced are generally acceptable for narrow or moderate bandwidth filters.

Appendix B
A Simplified Model for Coupled Cavities in Generalized Slot-Coupled Combline Filters

As explained in Section IIA, each cavity in the filter structure considered in this paper can be coupled at most with the other three cavities, as illustrated schematically in Fig. 8. The admittances $Y_{k,(O)}^t$ and $Y_{k,(S,i)}^t$ are defined by

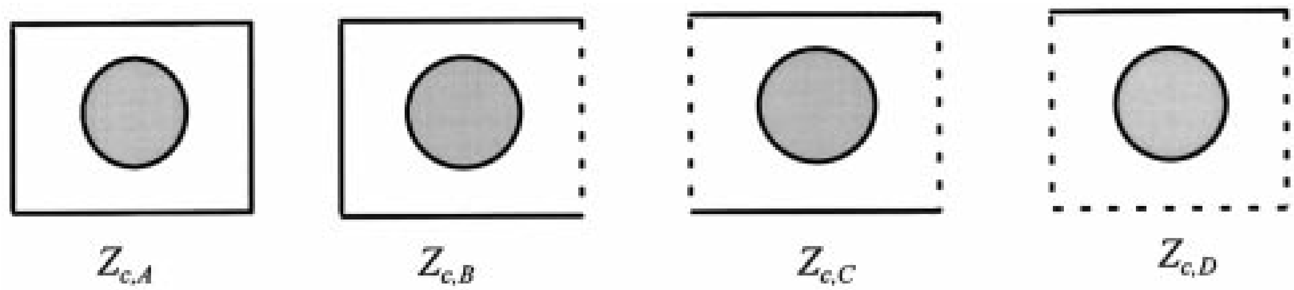

Fig. 10. Cross sections of the equivalent transmission lines in Fig. 9. The dotted lines represent either an electric or a magnetic wall, depending on the specific admittance ($Y^t_{k,(O)}$, $Y^t_{k,(S,i)}$) to be evaluated; the continuous lines are the actual metal boundaries of the cavity.

inserting electric and/or magnetic walls in the places of the three possible slots in the kth cavity. The evaluation of these admittances can then be performed (considering only the TEM mode) by means of the simple equivalent circuit reported in Fig. 9. This circuit is constituted by the cascade of four transmission-line sections, terminated with a short circuit at one side and with a capacitive susceptance $B_{T,k}$ (representing the tuning screw) at the other side. The admittances $Y^t_{k,(O)}$ and $Y^t_{k,(S,i)}$ are evaluated at the section of $B_{T,k}$. The equivalent transmission lines, whose lengths depend on the slots lengths as shown in Fig. 9, are of coaxial type with the inner conductor of circular shape (the resonator rod), the outer cross section (of rectangular shape) is constituted by electric and magnetic walls (at least one electric), depending on the slots' lengths and on the specific admittance ($Y^t_{k,(O)}$, $Y^t_{k,(S,i)}$) to be evaluated. With reference to the cavity in Fig. 8, a schematic representation of the equivalent transmission lines' cross sections is given in Fig. 10, where the continuous lines are the actual metal boundaries of the cavity and the dashed lines represent either electric or magnetic walls depending on the admittance to be evaluated.

To perform the computations with the equivalent circuit the characteristic impedances $Z_{c,A}$, $Z_{c,B}$, $Z_{c,C}$, and $Z_{c,D}$ of the transmission lines must be known. Their computation, however, can be made once at the beginning of the design procedure (in fact they do not depend on the design unknown, i.e., the slots lengths w_i). For designing the test filter these characteristic impedances have been numerically computed with a method based both on conformal transformation and on finite difference solution of Laplace equation [15].

In conclusion, it is worth remarking that the model presented here is suitable only for a first order design of slot-coupled combline filters (all high-order modes excited at the discontinuities determined by the slots and the tuning screws are discarded); it is, however, adequate to perform comparisons with different filter design approaches, as explained in Section III.

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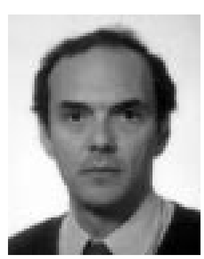

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